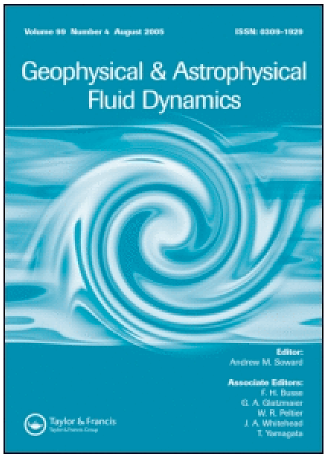


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# The intermittent solar cycle

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# THE INTERMITTENT SOLAR CYCLE

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A prominent feature of the solar cycle is the rise and fall of the number of sunspots on the surface with a timescale of approximately eleven years. The mathematical description of this behavior is complicated by the interruption of the cycle for 75 years starting around 1650. Similar previous intermissions of this kind are implied by the available data. We explore the possibility of modeling such temporal variations of the sunspot number with a deterministic dynamical system of relatively low order. The system we propose manifests on/off intermittency in which the cyclic variations of the solar activity switch off almost completely for extended periods. We also offer an explanation of the variation of the fluctuating part of the sunspot number over the cycle.

KEY WORDS: Intermittency, chaos, solar cycle.

## 1. INTRODUCTION

The variations in solar activity are called cyclic because there is a fairly regular recurrence of large numbers of large sunspots every eleven years or so. For many decades, it had been assumed that these spots form within the solar convection zone, which occupies the outer third of the sun. But there are difficulties with this point of view that are most easily avoided by assuming that the spots form elsewhere. One of these objections is that it is difficult to form a strong magnetic flux tube in the solar convection zone since it would be quickly buoyed up to the surface before the field in it could be raised to a very large value. This problem could be avoided by moving the seat of the solar cycle to just below the convection zone where the density is stably stratified (Spiegel and Weiss, 1980). Such a change in venue has been considered as a possibility by an increasing number of authors who variously place the origin of sunspots in the depths of the convection zone (e.g. DeLuca, 1986) or even lower down (e.g. Layzer *et al.*, 1979).

The possibility that solar activity could originate beneath the convection zone has received indirect support from helioseismology. Since a key ingredient of model solar dynamos is differential rotation, the discovery that solar differential rotation extends into a (seemingly) thin, shearing layer below the convection zone makes it reasonable to postulate dynamo action there. The existence of this layer, which has been renamed the tachocline, can be rationalized if it is assumed that it is anisotropically turbulent (Spiegel and Zahn, 1992). We shall not go into that issue here, and mention it only to suggest that the tachocline is likely to be turbulent and is a promising site for dynamo action. In particular, the stable density stratification in the tachocline is favorable for the buildup of strong fields.

In this rush to lower the seat of the solar cycle, we must not overlook the strengths of the early arguments placing the main solar dynamo in the convection zone itself. Indeed, we should expect considerable dynamo action in the convection zone, even though we may not expect that strong spots will form there. We believe that it is necessary to have dynamos in both the convection zone and the tachocline if the solar cycle is to be understood fully. In particular, it is in the interaction of these two processes that we shall seek the origin of the observed intermittency of the cycle.

Our picture of the origin of the solar intermittency is sufficiently simple that we can sketch it at the outset, prior to setting out details of the dynamical model. We start with the assumption that the cyclic character of the solar activity has its origin in a Hopf bifurcation that we attribute to an overstability arising in the tachocline. What the explicit cause of this instability may be is open, though magnetic buoyancy (Childress and Spiegel, 1981) and the overstability of an $\alpha - \omega$ dynamo (Proctor and Spiegel, 1991) have been considered as possibilities.

We shall moreover assume that, whatever the nature of the instability mechanism may be, the control parameter of the tachocline in isolation may be set to a value that produces stability. However, the tachocline is in a chaotic, or noisy, environment. It is coupled to the convection zone, which causes a renormalization of its stability parameter. Just as nonlinear terms in a dynamical system can saturate in linear growth, they can also overcome a linear damping and produce excitation. So, when the magnetic input from the solar convection is strong enough, the tachocline is driven through a Hopf bifurcation that produces cyclic activity, which may decay rapidly away when this input has a suitable fluctuation. This is a process that we have discussed in a general context elsewhere (Platt *et al.*, 1992) and that we call on/off intermittency. Its workings shall become explicit when we turn to the model below.

We should first explain that the model is really to be understood as a set of equations generating behavior very like that seen in the solar cycle. Though it was created from some physical picture of the process, that picture may now be forgotten as we attempt to see what physical processes may be read into the equations. In a similar way, the equations themselves represent just a simple version of the dynamical processes they exemplify.

## 2. THE DATA

Shortly after Galilei and his contemporaries established that spots were really on the surface of the sun, observation of these surprising objects began in earnest. This period opened with a dearth of spots that lasted for about seventy-five years from about 1650.

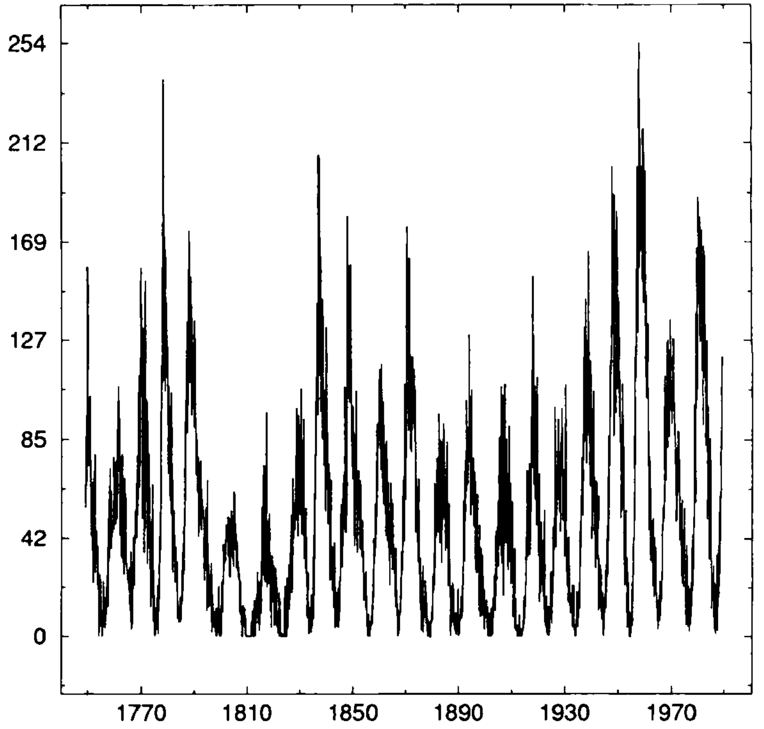
Reports of such intermissions (Maunder 1890, 1894) were long ignored in modern times until J. A. Eddy (1976) re-examined the evidence and concluded that it was real and significant. Eddy named the period of low sunspot number the Maunder minimum and discussed other lines of evidence suggesting that periods in which the solar cycle becomes dormant recur every few hundred years.

Even when these intermissions are overlooked, analyses of the continuous cycle are not all in accord. Mandelbrot and Wallis (1969) report that their analysis of the data reveals no periodic component and suggest that the solar cycle is best regarded as a $1/f$ process. On the other hand, the solar cycle has been described as a simple limit cycle (Gudzenko and Chetroprud, 1985; Ruzmaikin, 1981). A compromise between the apparent randomness of the former work, with the order of the latter, is provided by models of deterministic chaos. Chaos not only provides the right kind of mild aperiodicity that we see in the solar cycle, but it may also produce the intermittency signaled by the Maunder minimum (Spiegel, 1977). A number of chaotic interpretations of increasing specificity have been offered for the solar cycle (Ruzmaikin, 1981; Spiegel and Wolf, 1987; Weiss *et al.*, 1985; Morfil *et al.*, 1991) and Weiss (1990) has given a concise discussion of the main issues.

The daily sunspot number, provided by the Zurich Observatory until very recently, has very large fluctuations. Though one can easily see the cyclic behavior in it, it is not clear how much of the daily variation is caused by factors extraneous to our immediate goal of modeling the cyclic process. We shall adopt instead the monthly averages of the sunspot number as our basic data. These are plotted in Figure 1 as a function of time. The monthly sunspot number, which we designate as $N$, according to the Zurich definition, gives higher weight to groups of spots than to isolated spots. Although the sunspot number in Zurich has been regularly recorded for only about 100 years, the sunspot number of the prior hundred or so years is also included in Figure 1 as inferred from reports of sunspots in the literature (Waldmeier, 1961). Even if we accept these numbers as reliable, a run of two hundred years is not very much to go on, but we shall try to see what these data suggest.

Since the monthly average of the sunspot number shows strong fluctuations, we take a look at the smoother version of the data shown in Figure 2. This is a plot of the running eleven-month average of the sunspot number, $\bar{N}$. Then, in Figure 3, we compare the fluctuations of the sunspot number, $\Delta_N = N - \bar{N}$ vs time together with $\bar{N}$ itself. This figure shows that the fluctuations are largest when the sun is most active and are more numerous than we would expect from root-$N$ noise.

Our aim is to make a dynamical model of the solar cycle that will capture the qualitative features of these data. Such a model cannot be unique, but one that works at all will be helpful in unraveling the physics of the solar cycle. One feature that we read into these data is that there is a continuous dynamo process, revealed by the fluctuations around the yearly means and their correlation with the cyclic activity. We attribute this continuous process to dynamo activity in the convection zone. We suggest that its modulation is an observational effect caused by the use of the sunspot number as an indicator of activity and will explain this notion below.

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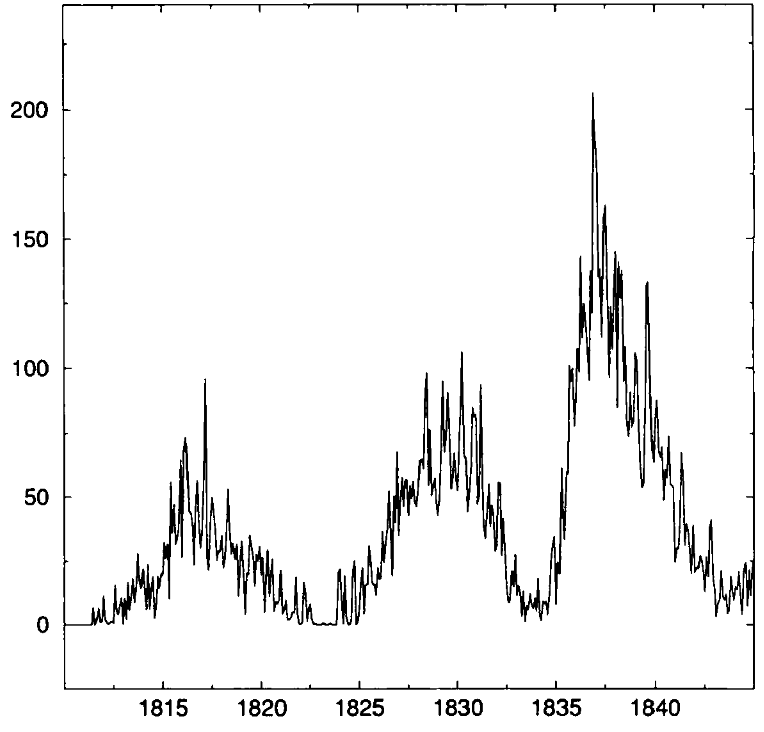


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**Figure 1** (a) Monthly average of the sunspot number $N$ for the last 250 years as reported by Zurich Observatory. (b) $N$ for three cycles.

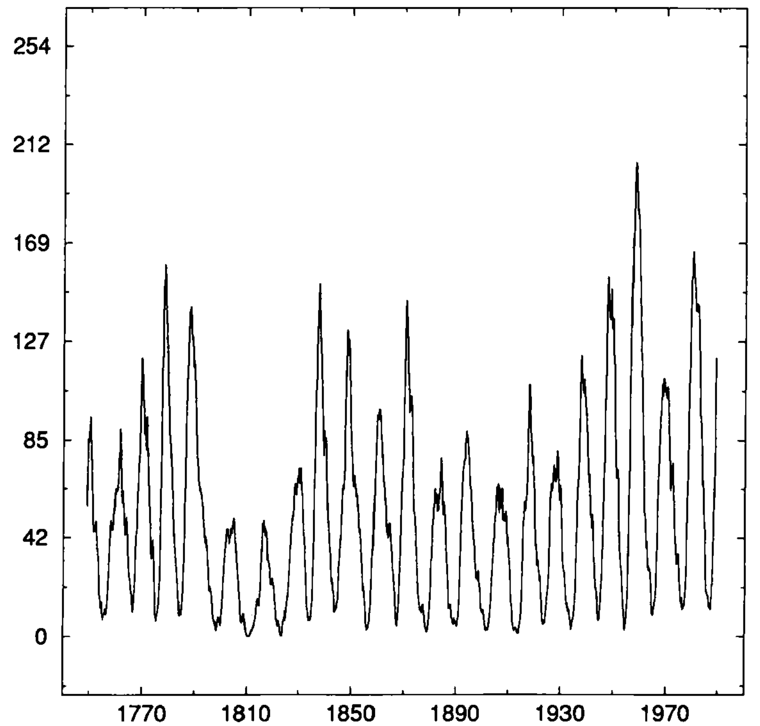



**Figure 2** Running eleven-month average $\bar{N}$ of the monthly mean sunspot number $N$.

## 3. THE MODEL

We propose a model based on two nearly independent dynamo processes. For the process producing the main solar cycle, we take a potentially overstable system. The normal form for such a system, assuming a supercritical Hopf bifurcation, is

$$\dot{x} = \beta x - \omega y - (x^2 + y^2)x, \tag{1}$$

$$\dot{y} = \omega x + \beta y - (x^2 + y^2)y. \tag{2}$$

We imagine this oscillator to be localized in the tachocline. The characteristic oscillation frequency, $\omega$, is associated to the eleven year cycle time. Unlike the time scale of a month, there is no obvious source for this one. It can be identified as the travel time of an excitation wave from midlatitude to the equator (Proctor and Spiegel, 1991), but that is not as clear cut an identification as we would like, so for now we simply think of $\omega$ as dictated by observation. A more subtle question, not directly answered observationally, is the choice of $\beta$, the growth rate, which we shall discuss presently.

The convection zone produces a conventional turbulent dynamo though, by conventional, we do not mean it is understood. For this process, we simply take the Lorenz equations, which model a shunted disk dynamo (Malkus, 1972) and have already been

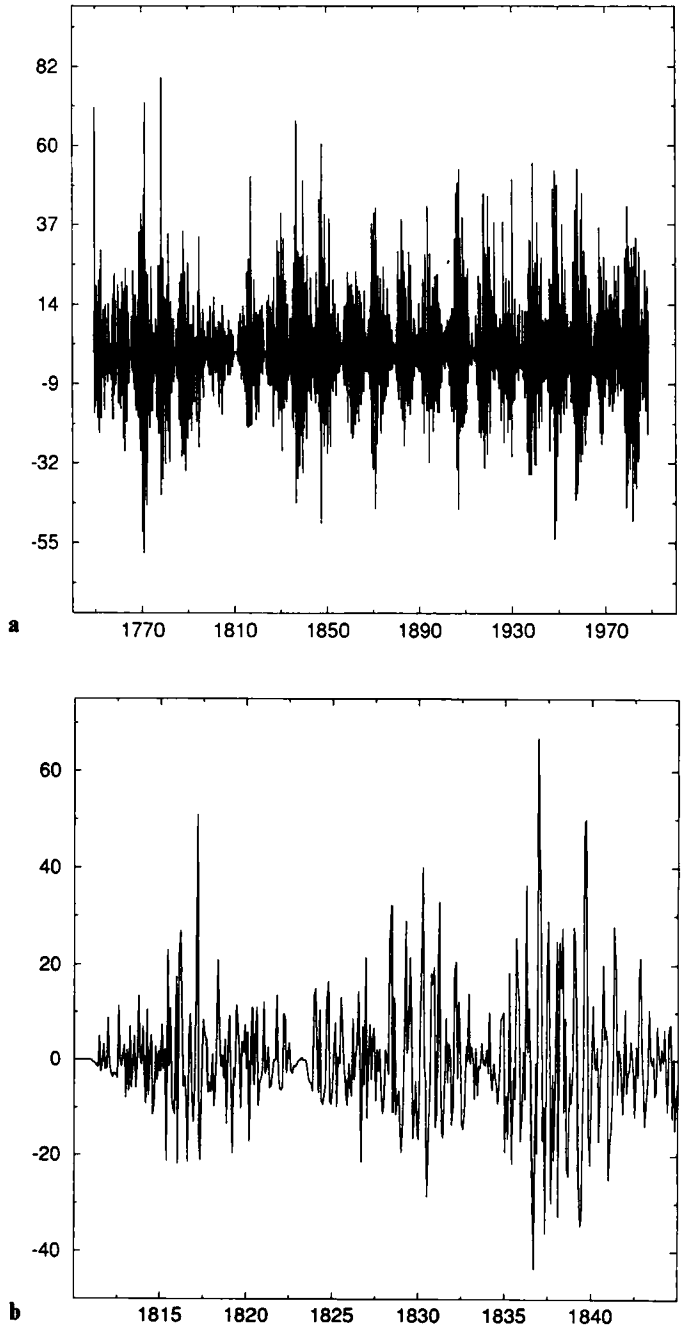



**Figure 3** (a) Fluctuations of the sunspot number $\Delta_N = N - \bar{N}$. (b) $\Delta_N = N - \bar{N}$ for three cycles. Comparison with Figures 1 and 2 shows that the envelope of the fluctuations is in-phase with the running eleven-month average $\bar{N}$.

mentioned in connection with the solar activity (Ruzmaikin, 1981). The Lorenz equations may be written in the form

$$\ddot{\xi} = a\xi^3 + b\zeta\xi - \mu\dot{\xi}, \tag{3}$$

$$\dot{\zeta} = -\varepsilon[\zeta + \alpha(\xi^2 - 1)]. \tag{4}$$

where $a$, $b$, $\mu$, $\varepsilon$ and $\alpha$ are parameters that we must choose. There is a great deal of freedom here, but we adopt values that make the system chaotic with a time scale that we identify with the deep convective time scale of one month.

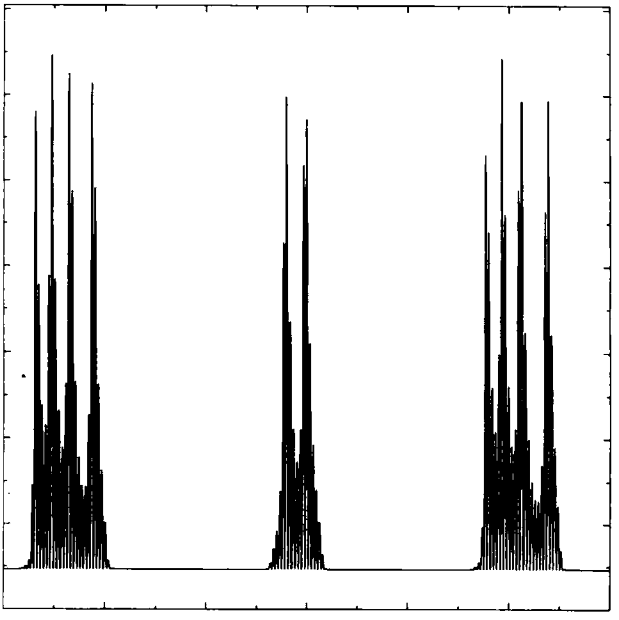
Now we need to decide what property of the solutions of these equations is to be called the theoretical sunspot number. The Zurich sunspot number is not simply a number of spots. It is ten times the number of spot groups plus the number of individual spots. This measure of solar magnetic activity is dominated by large spot complexes. In our picture, the large spot complexes are produced by the strong flux tubes that originate in the tachocline, so we shall give a similarly strong weighting to the output of (1)–(2).

We shall assume that one of the coordinates in each of (1)–(2) and (3)–(4) corresponds to the poloidal magnetic field, which is mainly responsible for making sunspots. It turns out not to matter much to which components we assign this role and we select $\xi$ and $x$. So the effective poloidal field is given by the pair of values $(x, \xi)$. The way we have set things up, this is a point in the product space of $x$ and $\xi$ and no coupling has been introduced between the two components. We could also include couplings but that makes it much harder to construct models with a desired behavior without a lot of searching through parameter space. In any case, we assume that the coupling between the tacholine and the monthly fluctuations from the convection zone is not significant.

We need to decide what the effective spot-making field is and we choose $x + 0.1\xi$, giving a greater weight to $x$ than $\xi$ as inspired by the Zurich weighting. But the sunspot number is always positive while this effective field strength changes sign. So we must introduce a relationship between theoretical spot number and the magnitude of the field variables. The simplest choice is to define the theoretical sunspot number

$$\mathcal{N} = (x + 0.1\xi)^2. \tag{5}$$

Now if we pick a constant, slightly positive, $\beta$, and run the two systems, we get a noisy looking but rather regular sunspot cycle, with no intermissions. It is not easy to produce the kind of on/off intermittency displayed by the solar cycle with simple second or third-order systems. There does exist a fifth-order system devised for the purpose (Spiegel, 1980; Platt, 1990), in which a chaotic oscillator drives a bifurcation parameter of a nonlinear oscillator that switches on and off in the desired manner. However, when it is on, it does not behave sufficiently like the sun. So we change the second order system to one which has a Hopf bifurcation, namely (1)–(2), and drive it with a chaotic third order system. This time, instead of the Lorenz driver of the previous fifth-order system, we use one with an astrophysical provenance (Moore and



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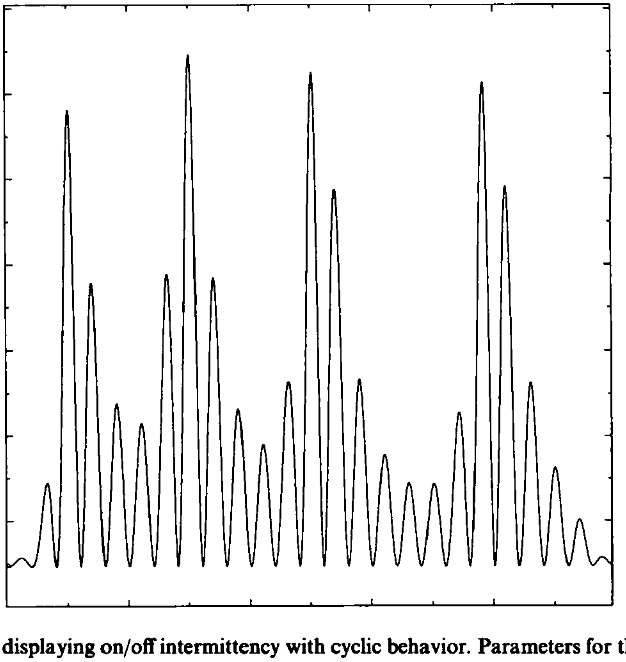

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**Figure 4** $x^2$ vs time displaying on/off intermittency with cyclic behavior. Parameters for the equations are $a = -b = 2.5E5$, $\mu = 845.625$, $\varepsilon = 205$, $\alpha = 6.5$, $\omega = 2$, $\beta = 1$, $X_0 = -0.15$, $A = 0.7$, $B = 0$, $\delta = 0.03$, $C = -0.5$. (a) shows a few of on/off cycles, (b) is a blowup of (a) showing cyclic behavior during an on cycle. Compare (b) with Figure 2.

a

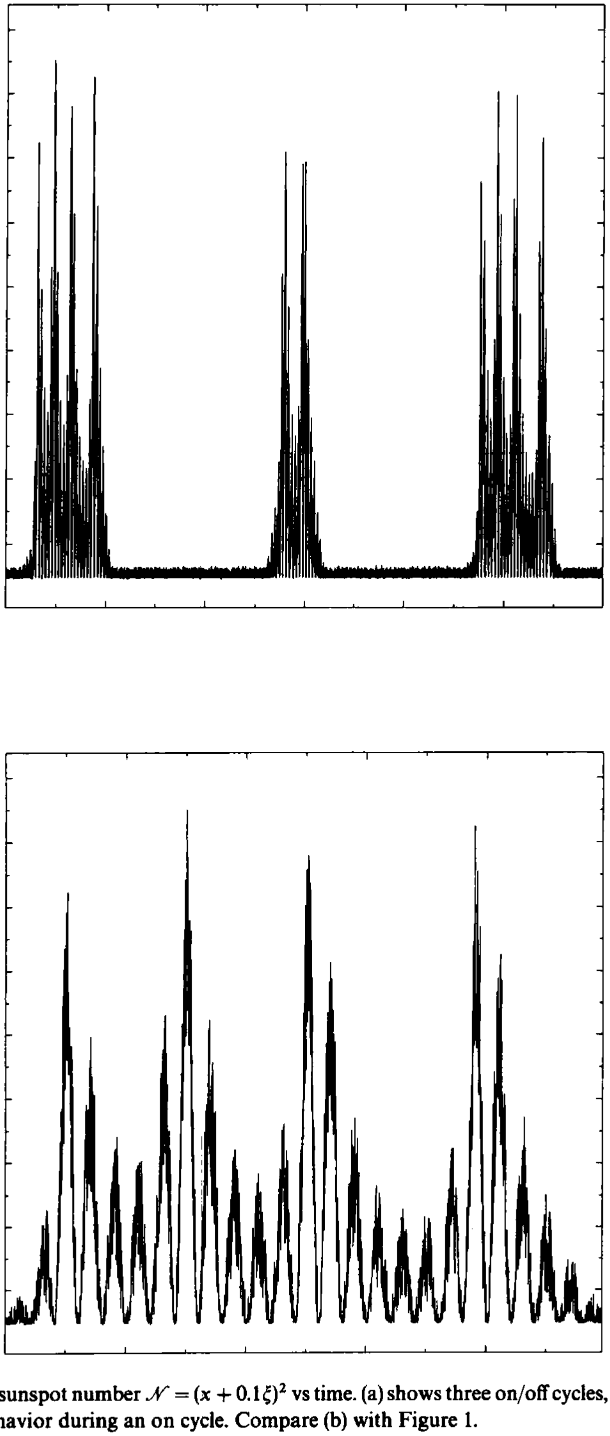

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**Figure 5** Simulated sunspot number $\mathcal{N} = (x + 0.1\xi)^2$ vs time. (a) shows three on/off cycles, (b) is a blowup of (a) showing cyclic behavior during an on cycle. Compare (b) with Figure 1.

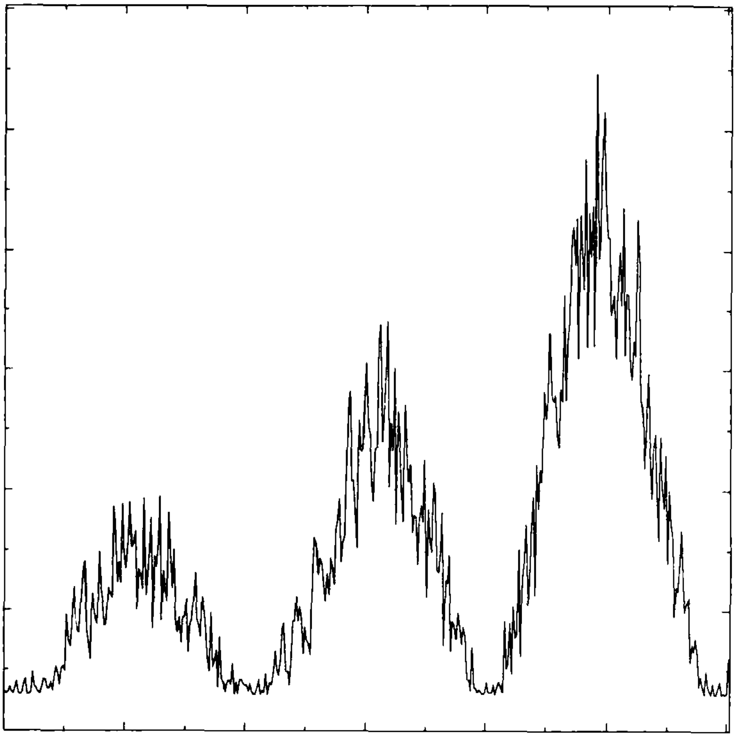


**Figure 6** Further blowup of Figure 5a showing three consecutive simulated cycles for comparison with Figure 1b.

Spiegel, 1966):

$$\ddot{X} = AX - X^3 - B\dot{X} + Z, \tag{6}$$

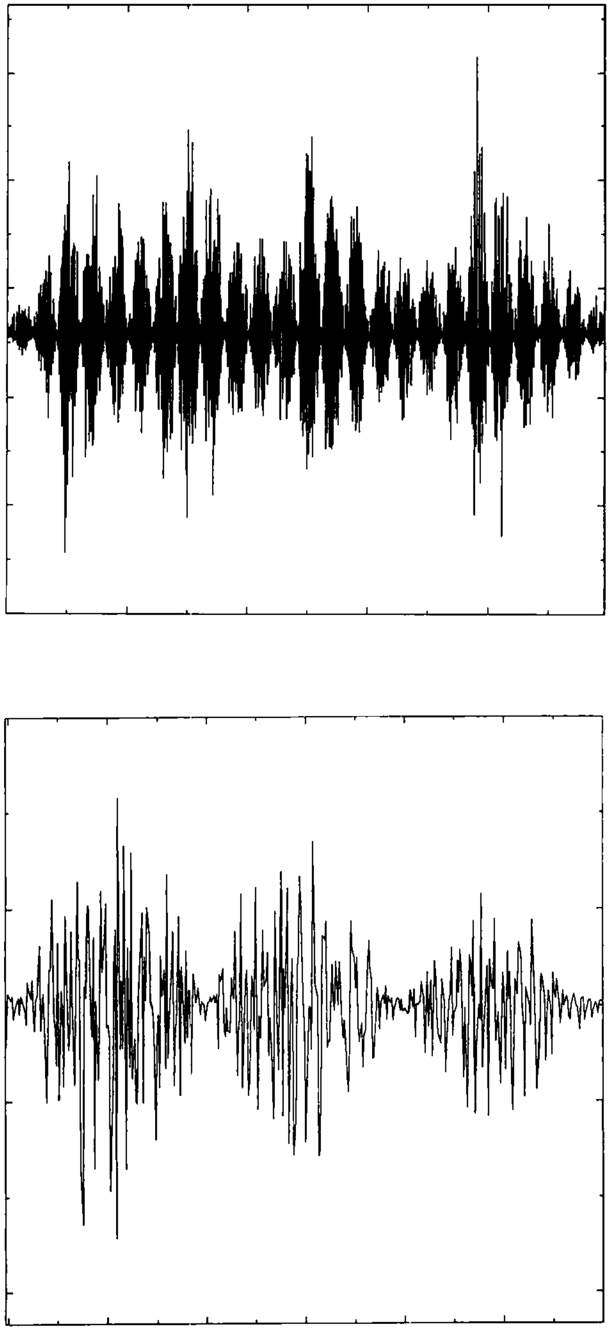
$$\dot{Z} = -\delta[Z - CX(X^2 - 1)]. \tag{7}$$

The parameter $\beta$ controls the instability of (1)–(2). The idea is to let

$$\beta = \beta_0(X - X_0), \tag{8}$$

where $\beta_0$ and $X_0$ are constants. The key parameter is $X_0$, for when $X$ is fixed at that value, an unending cycle is maintained.

Our motivation in setting up (6)–(7) is the picture that the tachocline generates strong flux tubes that float up to the solar surface where they protrude to make sunspots. To continue the process, it is necessary to replenish the field in the tachocline. This is done by the raining of magnetic flux down from the convection zone. The mean magnetic "rainfall" is measured by $X$ (or some function of it). In order to replenish the magnetic fields lost to make spots we need $X$ to be at least as large as the equilibrium



a

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**Figure 7** (a) Fluctuations of the simulated sunspot number $\Delta_N = \mathcal{N} - \bar{\mathcal{N}}$, where $\bar{\mathcal{N}}$ is computed by taking a running mean over eleven theoretical months. (b) is a belowup of (a) showing three consecutive simulated cycles. Comparison with Figures 5, 6 and 4b shows the correlation between these signals.

value $X_0$. In periods of magnetic "drought", with $X$ less than $X_0$, the cycle falters. We have elected to model $X$ as a chaotic process, though it could well have been a stochastic one for our purposes (Platt *et al.*, 1992). Moreover, we have called $X$ the variable rate of supply and $X_0$ the constant demand rate. But it might be more realistic to suggest that the convection zone supplies the field more or less steadily and it is the tachocline's state and magnetic requirements that are erratic.

The remaining issue is the choice of the parameters in (6)–(7). Again we set them so that the system is chaotic and then the only crucial thing is the characteristic time scale for this system, which is basically the characteristic time for $X$ to achieve values in excess of $X_0$. In effect, this is a two-parameter choice. We find that time scales that work can be longish, say a few cycle times. There is no evident reason why this is the characteristic time to achieve suitable fluctuations in the magnetic balance $X - X_0$. It may have to do with convective statistics and it happens to be roughly the time scale for thermohydrostatic adjustment of the tachocline (Spiegel, 1987). The key fact in all these parameter choices is that broad ranges of parameters work to make the right qualitative behavior. When we vary $X_0$, other things being equal, what is being adjusted is the duty cycle of the solar cycle. This feature of the model may be of interest in the study of activity cycles in other cool stars. Here we have tuned $\delta$ in (7) to make the recurrence of solar activity rather regular.

Turning to results, we display $x^2$ vs time in Figure 4 to illustrate the on/off behavior of the system. This is a system displaying on/off intermittency with cyclic behavior during the on period. Figure 5 is similar to Figure 4, but this time $\mathcal{N}$ is plotted instead of $x^2$ and it contains the fluctuations. Figure 6 is an even larger blowup than Figure 5a and it shows just three simulated cycles. It should be compared with a similar blow up to the monthly sunspot number of Figure 1b, also for three cycles.

## 4. DISCUSSION

We have modeled the temporal variation of the solar cycle as if it were the result of some magnetic weather system analogous to that of the earth. The solar convection zone is like the earth's atmosphere and the solar depths are like the ocean. The top layer of the earth's ocean, the thermocline, was the model for naming the solar tachocline. The tachocline is what some atmospheric scientists might call the weather layer and it produces the primary features of the observed solar cycle. It is described by equations (1) and (2), which we shall call system A, the solar oscillator.

The solar convection zone has tremendous magnetic activity and we have split this into two parts, like weather and climate. That is, we have described the magnetic fluctuations, assumed to occur on a convective time scale of one month, by the chaotic oscillator given in equations (3) and (4). Call that system B. We assume that we observe the effects of system A on the magnetic weather in the form of a magnetic signal, $\xi$. Meanwhile, there is a net exchange of magnetic flux between the convection zone and the tachocline that is like the net effects of evaporation and storms in the ocean-atmosphere system. The fluctuations of the mean magnetic exchange are too complex for us to really say much about at this stage of the subject. We have described it with ruin the skew product structure, but it is an example of how new small coupling terms can introduce nice nuances.

A significant modification in the earlier fifth-order system is that the second-order subsytem is a Hopf oscillator rather than a conventional nonlinear oscillator as in the earlier model. And, somewhat parochially, we have used a different chaotic oscillator than the Lorenz system. But, as before, we have two coupled oscillators as the basic structure with the chaotic oscillator playing a role analogous to the quasi-fixed point in Pomeau-Manneville intermittency.

The quasi-fixed point in the Pomeau-Manneville model, is a point of a map whose image is close to the point itself. Once the system gets close to this point, it lingers there for several iterations. Such a point in a map corresponds to a periodic orbit. In our on/off models we use invariant objects to control the behavior and work directly with flows. In the present instance we adopt a weakly unstable, chaotic attractor as the invariant object. It represents states near to which the combined system will tend to spend long times, and it organizes the intermittent behavior. The organizing object does not need to be invariant; it suffices that it be quasi-invariant like the fixed point in the Pomeau-Manneville case, so that systems that enter their neighborhoods remain there for a long time. Systems constructed along these lines will produce behavior qualita-tively like that seen in the solar cycle, and our explicit example illustrates this. We do not insist upon any of the details. Our aim has been to capture some of the prominent features of the solar cycle in a relatively simple dynamical system.

Having constructed a mathematical model of the solar behavior, we may now consider physical processes that it can describe. But we think it best to first generalize the model to include spatio-temporal behavior. If the Hopf bifurcation occurs in a thin layer like the tachocline, then we shall have many modes passing through the Hopf bifurcation nearly at once. For a packet of such modes, we should replace system A by a suitable partial differential equation (Proctor and Spiegel, 1991). Our next step must be to endow this richer system with on/off intermittency. On the longer time scale, we must contemplate the derivation of such equations in detail from models of the convection zone itself. This will open the way to more quantitative comparison of theory and observation.

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## *References*

Childress, S. and Spiegel, E. A., "A Prospectus for a Theory of Variable Variability," in: *Variations of the Solar Constant* (Ed. S. Sofia) NASA Conference Publication 2191, Goddard Space Flight Center, Greenbelt, Md., pp. 273–292 (1981).

Deluca, E. E., "Dynamo Theory for the interface between the Convection Zone and the Radiative Interior of a Star," *Thesis*, (U. of Colorado), NCARCT–104 (1986).

Eddy, J. A., "The Maunder Minimum," *Science* **192,** 1189–1202, (1976).

equations (6) and (7), another chaotic oscillator. This system, which we call system C, describes the long time fluctuations in the net magnetic exchange between the tacholine and the convection zone. We liken C to a climate system in that it involves relatively slow variations.

The two systems that are assumed to produce directly observable effects, A and B, are completely uncoupled in our model. At first sight, this is surprising, since their analogues in the observations, the fluctuations and the cycling in the sunspot number, are manifestly correlated, as we saw in Figure 3. But our theoretical output also has such a correlation. In Figures 5–7 we show the theoretical sunspot number $\mathcal{N}$ and its fluctuation $\mathcal{N} - \bar{\mathcal{N}}$ where $\bar{\mathcal{N}}$ is computed by taking a running mean over eleven theoretical months. The reason for this correlation is that, although the outputs of the two systems add linearly, the theoretical sunspot number given in equation (5) is a nonlinear function of that combination. Hence there is a cross term in the computed signal that makes a correlation between $\mathcal{N}$ and $\mathcal{N} - \bar{\mathcal{N}}$.

Whether the correlation between the observed sunspot number and its fluctuations is also to be explained in this simple way could be tested by examination of magnetograms. If the answer were to be that simple, we need not worry about the possible coupling terms between systems A and B. We have kept such coupling to a minimum to simplify the models and to reduce the searching required to find optimal parameter values. By leaving A and B uncoupled, we are easily able to understand each component separately and choose parameter values quite easily.

In fact, system B is really icing on the cake. We included it because the cyclic behavior of the fluctuations in $\mathcal{N}$ loomed as a possibly significant effect. Having seen that there exists a simple possible explanation (that remains to be tested) we are less concerned with the fluctuations and more interested in the main event, the cycle and its intermittent behavior, as produced by systems A and C. The coupling of these two systems is a more interesting affair.

In the fifth-order system that was a precursor to A + C (Spiegel, 1980; Platt, 1990), there was strong coupling between the two components. In the new A + C, we let C influence A, but do not allow A to feed back on C. This permits us to visualize much more easily what the model will do and to avoid a lot of wandering through parameter space with the computer. This device of one-way coupling is commonly used in physics and mathematics. In thermodynamics, a thermal bath affects the system of interest without feeling it. In geophysical fluid dynamics, the $1\frac{1}{2}$-layer system has just such a one-way interaction. Mathematicians call a system with such one-sided dynamics a *skew-product structure*. Its value for analysis is manifest and we have endowed the present A + C system with such a structure for this version of the model.

Having seen the general features of the model in this limit, we can add couplings and see what effects they may have. We shall not do that here, but prefer to wait to see what further observational details need modeling. However, we may mention an example of such extras. There seems to have been a weak cyclic activity during the Maunder minimum and Ribes (Zahn, personal communication) has reported that the published data support this. If we introduce a term proportional to $X$ in equation (1) we get such an effect. Its relative importance depends on such details as the choice (5). Thus, if we use $|x + 0.1\xi|$ instead of (5) we get a more pronounced effect. Of course, this does not

Gudzenko, L. I. and Chetroprud, V. E., "Some Dynamical Properties of the Cyclic linear Dynamos: a Complex Generalization of the Lorenz Equations," *Physica D* **14**, 161–176 (1985).

Layzer, D., Rosner, R. and Doyle, H. T., "On the origin of Solar Magnetic Fields," *Astrophys. J.* **229**, 1126–1137 (1979).

Malkus, W. V. R., "Reversing Bullard's Dynamo," *Eos, Trans. Amer. Geophys. Un.* **53**, 671–671 (1972).

Mandelbrot, B. B. and Wallis, J. R., "Some Long-Range Properties of Geophysical Records," *Water Resources Research* **5**, 321–340 (1969).

Maunder, E. W., "Professor Sporer's research on sunspots," *Mon. Not. R. astr. Soc.* **50**, 251–252, (1890).

Maunder, E. W., "A Prolonged Sunspot Minimum," *Knowledge* **17**, 173–176 (1894).

Moore D. W. and Spiegel, E. A., "A Thermally Excited Nonlinear Oscillator," *Astrophys. J.* **143**, 871–887 (1966).

Morfill, H., Scheingraber and Sonnett, C. P., "Sunspot Number Variations: Stochastic or Chaotic," in: *The Sun Time* (Eds. C. P. Sonnett, M. S. Giampapa and M. S. Matthews) U. of Arizona press, pp. 30–58 (1991).

Platt, N., "Behavior of a Fifth Order System of ODE's with Intermittency," in: *Proceedings, GFD Summer Program* (R. Salmon, dir.) Woods Hole Oceanographic Institution, WHOI-91-93, 320-335, (1990).

Platt, N., Spiegel, E. A. and Tresser, C., "On-Off Intermittency: a Mechanism for Bursting," *Phys. Rev. Lett.* **70**, pp. 279–282 (1992).

Proctor, M. R. E. and Spiegel, E. A. "Waves of Solar Activity," in: *The Sun and the Cool Stars* (Eds. D. Moss, G. Rüdiger and I. Tuominen) Springer-Verlag, 117–128 (1991).

Ruzmaikin, A. A., "The Solar Cycle as a Strange Attractor," *Comm. Astrophys. Space Phys.* **9** 85–93 (1981).

Spiegel, E. A., "Introduction," in: *Problems in Stellar Convection* (E. A. Spiegel and J.-P. Zahn) Springer-Verlag, Berlin 'Heidelberg' New York, p. 1 (1977).

Spiegel, E. A., "A Class of Ordinary Differential Equations with Strange Attractors," in: *Nonlinear Dynamics* (Ed. R. H. G. Helleman) *Ann. N.Y. Acad. Sci.*, **617**, 305–312 (1980).

Spiegel, E. A., "Hydrostatic Adjustment Time of the Solar Subconvective Layer," in: *The Internal Solar Angular Velocity* (Eds. B. R. Durney and S. Sophia) pp. 321–328 (1987).

Spiegel, E. A., and Weiss, N. O., "Magnetic Activity and Variations in Solar Luminosity," *Nature* **287**, 5783–5784, (1980).

Spiegel, E. A. and Wolf, A. N., "Chaos and the Solar Cycle," in: *Chaotic Phenomena in Astrophysics* (Eds. J. R. Buchler and H. Eichhorn) *Ann. N.Y. Acad. Sci.* **497**, 55–60, (1987).

Spiegel, E. A. and Zahn, J.-P., "The solar tachocline," *Astron. Astrophys.* **265**, 106–114 (1992).

Waldmeier, M., The Sunspot-Activity in the years 1610–1690, Schulthess, Zurich (1961).

Weiss, N. O., Cattaneo, F. and Jones, C. A., "Periodic and Aperiodic Dynamo Waves," *Geophys. Astrophys. Fluid Dynam.*, **30**, 305–341 (1984).

Weiss, N. O., "Periodicity and aperiodicity in solar magnetic activity," *Phil. Trans. R. Soc. Lond. A* **330**, 617–625 (1990).