

Physics 8012
Problem set 3, due on 9/24/08
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Here, you are asked to derive a few properties of the isothermal lens, which has a density profile of $\rho = \frac{\sigma^2}{2\pi Gr^2}$.

- a. Work out the deflection angle and show that indeed it has a constant magnitude. In the class, we actually wrote down an integral expression for it. You can either integrate this directly, or you might want to try the trick of differentiating (taking the 2D divergence of α^i) and then integrating back (hint: think about basically 2D electrostatics).
- b. Write down the lens equation and work out the position of the images. Under what condition would you find more than one image? What is the typical size of the Einstein ring in arc-seconds? (A typical galaxy has $\sigma \sim 300$ km/s, and typical lens/source distances are cosmological, which means about 1 Gpc. For your information, $1 \text{ pc} \sim 3 \times 10^{18}$ cm.)
- c. Work out the distortion matrix and find the image magnification for each image.