Physics 8012
Problem set 4, due on 10/6/08
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1. In class, we have shown that in a flat universe the lensing convergence is

$$\kappa = \int_0^{\chi_S} d\chi \frac{(\chi_S - \chi)\chi}{\chi_S} \nabla_\perp^2 \phi \tag{1}$$

which we rewrote as

$$\kappa = \int_0^{\chi_S} d\chi \frac{(\chi_S - \chi)\chi}{\chi_S} \left(\nabla^2 \phi - \frac{d^2\phi}{d\chi^2} \right) \tag{2}$$

where $d/d\chi$ is a total derivative and we have ignored time derivatives of ϕ. Show that the above can be further approximated by

$$\kappa = \int_0^{\chi_S} d\chi \frac{(\chi_S - \chi)\chi}{\chi_S} \nabla^2 \phi \tag{3}$$

You might find the following approximation useful: $\nabla_\perp^2 \phi \gg (d\phi/d\chi)/\chi$, which basically says that the scale on which ϕ varies is typically much smaller than χ (χ here is on cosmological scale i.e. of the order of χ_S). (Also note: this approximation of course breaks down when χ is close to zero, but you will see that the relevant integral receives relative little contribution from small χ's.)

2. Using the Fourier transform

$$\kappa(\boldsymbol{\theta}) = \int \frac{d^2\ell}{(2\pi)^2} \kappa(\boldsymbol{\ell}) e^{i\boldsymbol{\ell}\cdot\boldsymbol{\theta}} , \tag{4}$$

show that the E mode we defined in class

$$\kappa(\boldsymbol{\ell}) = \cos 2\beta \gamma_1(\boldsymbol{\ell}) + \sin 2\beta \gamma_2(\boldsymbol{\ell}) \tag{5}$$

gives

$$\kappa(\boldsymbol{\theta}) = B \int d^2\theta' \frac{1}{|\boldsymbol{\theta} - \boldsymbol{\theta}'|^2} \left[\cos 2\alpha \gamma_1(\boldsymbol{\theta}') + \sin 2\alpha \gamma_2(\boldsymbol{\theta}') \right] \tag{6}$$

and find the constant B.

Clarification on some of the notations: I use the arguments to denote Fourier space versus real space quantities i.e. $\boldsymbol{\theta}$ for real space and $\boldsymbol{\ell}$ for Fourier space; β is the angle between $\boldsymbol{\ell}$ and the x-axis; α is the angle between $\boldsymbol{\theta} - \boldsymbol{\theta}'$ and the x-axis.