Physics 8012
Problem set 5, due on 10/20/08
Lam Hui

1. In class, we have used the geodesic deviation equation to compute the response of test masses to gravitational waves. You can find a derivation of the geodesic equation in Carroll. Here, let's derive it in a more direct way a la Weinberg (though if read Weinberg, you have to be careful Weinberg's sign convention for various quantities differs from Carroll's).

Recall that the geodesic deviation equation is

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu{}_{\nu\lambda}(x) \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} \tag{1}$$

where τ is the affine parameter (or proper time along the trajectory of a massive particle), and $\Gamma^\mu{}_{\nu\lambda}$ is the affine connection.

A nearby geodesic obeys the following equation

$$\frac{d^2(x^\mu + \delta x^\mu)}{d\tau^2} + \Gamma^\mu{}_{\nu\lambda}(x + \delta x) \frac{d(x^\nu + \delta x^\nu)}{d\tau} \frac{d(x^\lambda + \delta x^\lambda)}{d\tau} \tag{2}$$

Note how I write out the argument of the affine connection explicitly in the above two expressions: one is evaluated along $x(\tau)$; the other is evaluated along $x(\tau) + \delta x(\tau)$.

Subtract these two equations, keep only first order terms in δx, and using the definition of the Riemann tensor, derive the geodesic deviation equation:

$$\frac{D^2 \delta x^\mu}{D\tau^2} = R^\mu{}_{\nu\rho\sigma} \frac{dx^\nu}{d\tau} \frac{dx^\rho}{d\tau} \delta x^\sigma \tag{3}$$

where $DA^\mu / D\tau$ is defined to be $\frac{dx^\nu}{d\tau} \nabla_\nu A^\mu$ where ∇_ν is the covariant derivative.

2. Starting from the E & M equation

$$\partial^\nu F_{\mu\nu} = 0 \tag{4}$$

with $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$. Impose the appropriate gauge condition and work out the appropriate wave solutions, just like what we did for GR. You should be able to show that the wave travels at the speed of light, that it has two (not three or four) polarizations, and that they are transverse.