Physics G8099/V3400: String Theory
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Problem Set 6, due March 13, 2008

1. Derive the closed string spectrum: tachyon, graviton, dilaton and axion. Refer to the notes for how to proceed. You can assume $A = -1$ for both left and right movers. You should be able to show (1) the tachyon has a mass of $m^2 = -4/\alpha'$; (2) states with only one internal oscillator e.g. $\alpha^\mu{}_{-1}|k\rangle$ or $\tilde{\alpha}^\mu_{-1}|k\rangle$ are not allowed; (3) states with one right and one left mover have zero mass: $\alpha^\mu{}_{-1}\alpha^\nu{}_{-1}|k\rangle$; (4) the suitable conditions on the polarization tensor $e_{\mu\nu}$ are such that there are only $(D-2)^2$ true distinct degrees of freedom. You can consult Polchinski 4.1 if you wish.

2. Justify why $: e^{ikX} :$ is the right vertex operator for the tachyon i.e. $|k\rangle$. Here, I suppress the summation over the μ index as usual. The colon refer to normal ordering in the sense of subtracting expectation value i.e.

$$: e^{ikX} :\equiv e^{ikX} - \langle e^{ikX}\rangle \tag{1}$$

We expect by definition

$$\langle ...|p^\mu|k\rangle = k^\mu\langle ...|k\rangle \tag{2}$$

For $: e^{ikX} :$ to be a good operator that corresponds to $|k\rangle$, it better be true that

$$\langle p^\mu : e^{ikX(0,0)} : ...\rangle = k^\mu\langle : e^{ikX(0,0)} : ...\rangle \tag{3}$$

where $\langle\rangle$ can be understood as an average over the path integral and ... denotes other operators (or more precsely, funtionals). Here we assume the tachyon vertex operator is at the origin.

Let's use the closed string expression

$$p^\mu = \left(\frac{2}{\alpha'}\right)\oint \frac{dz}{2\pi}\partial X^m u(z) \tag{4}$$

The above is a contour integral. Let's think of the contour as a little circle around the origin. We will assume the other operators ... are at the far infinity so that we don't have to worry about poles around them when doing the above contour integral. (It is possible to relax this assumption, and then one has to think about more commutators. Let's not worry about this, everything would work out if done carefully.)

You can think of the tachyon vertex operator as

$$: e^{ikX} := 1 + ik_\alpha X^\alpha - \frac{1}{2}k_\alpha k_\beta[X^\alpha X^\beta - \langle X^\alpha X^\beta\rangle] + \text{higher} \quad \text{order} \quad \text{terms} \tag{5}$$

If you can show Eq. (3) holds ignoring the 'higher order terms', that would be good enough, although you should be able to see how the equation might hold even if you include these higher terms.