

Science 003B
Problem Set 3, due 2 July 2008
Lam Hui

1. *a.* Suppose the equation of state of some form of substance is $P = w\varepsilon$, where w is a constant. Derive from the first law of thermodynamics, as discussed in class, the relation between ε and a .
b. Repeat the same calculation, but now assume $w = w_0 + w_1 a$, where w_0 and w_1 are constants.

2. Derive from the Friedmann equation and the first law of thermodynamics the following expression:

$$\ddot{a} = -\frac{4\pi G a}{3c^2}(\varepsilon + 3P) \tag{1}$$

You should be able to derive the above expression without assuming flatness. This expression tells us under what condition we would have accelerated expansion. What is the condition? If $P = w\varepsilon$ as in question 1, what is the condition on w in order to have accelerated expansion?

3. Given that $\Omega_m \sim 0.3$ and $\Omega_r \sim 10^{-4}$ today, at what redshift z did our universe make the transition from radiation dominated to matter dominated?