

Science 003B
Problem Set 4, due 4 July 2008
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Here, you are to derive the (spatial) metric for the three-dimensional closed universe, by embedding it in four-dimensional space. Let us start with a four-dimensional space with a metric of the form $ds^2 = dx^2 + dy^2 + dz^2 + dw^2$. Consider a 3-sphere that satisfies the condition $R^2 = x^2 + y^2 + z^2 + w^2$. Show that the metric on the 3-sphere is given by $ds^2 = dr^2 + R^2 \sin^2(r/R)[d\theta^2 + \sin^2\theta d\phi^2]$. You will find it useful to refer to the expressions given in class that relate x , y and z to r , θ and ϕ . Furthermore, work out the area of the 2-sphere at $r = r_*$ according to this metric (i.e. this is the 2-sphere with r at the constant value r_* , and θ spanning 0 to π and ϕ spanning 0 to 2π .) What is the area in the limit of $R \rightarrow \infty$? Throughout this problem, you can ignore time (everything is purely spatial).