Science 003B
Problem Set 5, due 11 July 2008 at 5 pm
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This is a do-it-yourself problem set, which means you are asked to do this problem set entirely on your own, without talking to anybody. You are however free to consult your lecture notes and any books you wish. Please hand in the completed problem set to Williamson Lam at Room 4583, the latest by 5 pm on Friday, 11 July 2008.

1. Suppose nucleosynthesis results in only the hydrogen nuclei (^{1}H or, in short, H) and the ^{4}He nuclei. Suppose further that the formation of ^{4}He is an instantaneous process, and that right before ^{4}He formation, the neutron to proton ratio (the number density of neutrons divided by the number density of protons) is 1/7. What is the resulting mass fraction of ^{4}He? The mass fraction is defined to be the total mass in ^{4}He divided by the total mass in both H and ^{4}He. In this problem, you can treat the proton mass and the neutron mass as approximately equal.

2. Consider the spatial metric (no time) for the 3-sphere: $ds^2 = dr^2 + [R\sin(r/R)]^2[d\theta^2 + (\sin\theta)^2 d\phi^2]$. Find the total volume of the 3-sphere. The interesting point is that the 3-sphere has a finite volume, unlike 3D flat space.

3. Suppose we live in a flat expanding universe filled only with a mysterious substance (let's call it X) which has an equation of state $P = -(2/3)\varepsilon$.

a. How should ε vary with the scale factor a?

b. How should the scale factor a depend on time t?

c. Suppose in this universe, the Hubble parameter today is observed to be H_0. Express the age of the universe today in terms of H_0.

4. In class, we rewrote the Friedmann equation as:

$$1 = \frac{8\pi G\varepsilon}{3c^2H^2} - \frac{k}{a^2H^2} = \Omega + \Omega_k \qquad (1)$$

Here Ω is supposed to denote the sum of Ω_m, Ω_r and Ω_Λ. Show that in the early universe, $\Omega \gg \Omega_k$, and therefore by the above equation, $\Omega \to 1$ and $\Omega_k \to 0$. Let's further make this concrete by working out one example. Let us assume the universe is actually not flat – for instance let's say $\Omega_k = 0.1$ today. Calculate Ω_k at the time of nucleosynthesis. As one of the intermediate steps, you need to figure out the scale factor a at the time of nucleosynthesis, assuming nucleosynthesis occurs when the temperature of the universe multiplied by the Boltzmann constant is about 0.1 MeV. (Interesting side remark: the fact that the universe is so fantastically flat at the beginning is known as the flatness problem – why should the universe start out so flat? Inflation, the theory we discussed in class, provides a solution to this problem.)