Physics 3002, Problem Set 11, due 4/29/09

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1. This is based on Ryden problem 9.4. Find the luminosity distance d_L to the surface of last scattering. You can assume the redshift of the last scattering surface is $z = 1100$. First, write down an exact expression for d_L for the Benchmark Model (i.e. a flat universe with $\Omega_{m,0} = 0.3$ and $\Omega_{\Lambda,0} = 0.7$). The exact expression would involve an integral that is hard to do. Don't bother doing it. Second, let's approximate the integral by putting $\Omega_{m,0} = 1$ and $\Omega_{\Lambda,0} = 0$. This time, you should be able to do the integral and find d_L in Mpc (assuming $H_0 = 70$ km/s/Mpc).

2. In class, we introduced the idea of optical depth τ which gives us the probability that a photon is scattered as it travels through a gas of electrons. We wrote it as

$$\tau = \int \frac{d\ell}{1/(n_e \sigma_e)} \tag{1}$$

where n_e is the (proper) number density of electrons, σ_e is the Compton scattering cross-section, and $d\ell$ is an infinitesimal (proper) distance that the photon traverses. The rationale for this expression is that $1/(n_e\sigma_e)$ is the mean free path of the photon. Therefore, distance divided by mean free path gives us the probability of scattering. Let us rewrite this by noticing $d\ell = cdt$ because the photon travels at the speed of light c. Therefore, we have

$$\tau = \int n_e \sigma_e c dt \tag{2}$$

where the limits of integration should be from t_e (some early time) to t_0 (today). Show that this can be rewritten as

$$\tau = \int_0^{z_e} n_e \sigma_e c \frac{dz}{(1+z)H(z)} \tag{3}$$

where z_e is the redshift associated with t_e. Ryden argued in equation 9.33 and 9.34 that $n_e\sigma_e c$ is proprtional to $X(z)(1+z)^3$, where $X(z)$ is the ionized fraction (i.e. fraction of protons/electrons that are in free form). Can you explain where the factor of $(1+z)^3$ come from?

Note that, strictly speaking, τ can be interpreted as a probability only if $\tau \ll 1$. Clearly, the above expression for τ can become large if for instance σ_e is very large, or if n_e is very large, in which case τ could exceed one, and it would make no sense to think of τ as a probability. What would be the correct expression for the probability of scattering in that case? (Hint: it is an expression that involves τ, and it reduces to τ in the small τ limit.)