Physics 3002
Problem Set 6, due 3/25/09

Lam Hui

1. Ryden problem 6.4.

2. Ryden problem 3.3.

3. In class, we have derived the (spatial) metric for the two-dimensional closed universe (known as a 2-sphere), by embedding it in three-dimensional space (basically thinking of the 2-sphere as the surface of a ball). Here, you are to derive the (spatial) metric for the three-dimensional closed universe, by embedding it in four-dimensional space. Let us start with a four-dimensional space with a metric of the form $ds^2 = dx^2 + dy^2 + dz^2 + dw^2$. Consider a 3-sphere that satisfies the condition $R^2 = x^2 + y^2 + z^2 + w^2$. Show that the metric on the 3-sphere is given by $ds^2 = dr^2 + R^2 \sin^2(r/R)[d\theta^2 + \sin^2\theta d\phi^2]$. The relation between x, y, z, w is $x = R\sin(r/R)\sin\theta\cos\phi$, $y = R\sin(r/R)\sin\theta\sin\phi$, $z = R\sin(r/R)\cos\theta$, $w = R\cos(r/R)$. What are the natural ranges for r, θ and ϕ so that the 3-sphere is covered exactly once? (You might find it helpful to review the natural ranges of coordinates on the 2-sphere.)