Physics 3002, Problem Set 8, due 4/8/09
Lam Hui

1. Recall from class that we wrote down an expression for the radial comoving distance to an object that emitted a photon at redshift z_e:

$$r = \int_0^{z_e} \frac{c dz}{H(z)} \tag{1}$$

where $H(z)$ is the Hubble parameter at redshift z. For low z's, we can Taylor expand $1/H(z)$ around $z = 0$ as:

$$\frac{1}{H(z)} = \frac{1}{H_0} - \frac{1+q_0}{H_0} z + ... \tag{2}$$

Show that q_0 is given by

$$q_0 = -\frac{a\ddot{a}}{\dot{a}^2} \tag{3}$$

where the expression should be evaluated at $z = 0$ (or equivalently, $a = 1$). This should involve no more than taking derivative of $1/H(z)$. Further, show that

$$q_0 = \frac{1}{2} \sum_i (1 + 3w_i)\Omega_{i,0} \tag{4}$$

where i stands for the different components of the universe. For instance, i could stand for radiation in which case $w_i = 1/3$, or it could stand for matter in which case $w_i = 0$, or it could stand for the cosmological constant in which case $w_i = -1$. Here, w_i is the equation of state of each component i.e. $w_i = \epsilon_i/P_i$ where ϵ_i is the energy density and P_i is the pressure of that component. And $\Omega_{i,0}$ gives the Ω today for each component i. To derive this equation, you will find it useful to use a corollary that we have proven earlier in class, which gives us an expression for $\ddot{a}$.

Finally, substitute equation (2) into (1) to show that at low z_e,

$$r \sim \frac{c}{H_0} z_e - \frac{c}{H_0} \frac{1+q_0}{2} z_e^2 \tag{5}$$

The first term on the right hand side gives exactly the Hubble law (that distance r is proportional to redshift z_e). The second term tells us the deviation from Hubble law once z_e becomes large enough.

2. This is slightly modified from Ryden problem 7.6. People observe quasars at high redshifts - these are very bright objects which are believed to be powered by massive black holes. Suppose you observe a quasar at a redshift of $z = 5$, and determine that the observed flux of the light from the quasar varies on a time scale of $\delta t_0 = 3$ days. If the observed variation in flux is due to a variation in the intrinsic luminosity of the quasar, what was the variation timescale δt_e at the time the light was emitted? (Hint: recall our derivation in class for the redshift, using the null geodesic condition.) For the light from the quasar to vary on a timescale δt_e, the bulk of the light must come from a region of physical size $R \leq R_{\max} = c\delta t_e$. What is $R_{\max}$ for the observed quasar?