Extrasolar Planet Detection

1 Extrasolar planet detection

Materials

graph of velocity versus time for 51 Peg, graph of relative flux versus time for HD209458b

Instructions

51 Pegasi, in the constellation Pegasus, was the first star found to have a planet orbiting it. The planet cannot be seen directly, even with the worlds best telescopes. Instead is was detected indirectly through its gravitational influence on the star around which it orbits. The planet tugs on 51 Peg as it orbits, moving the star slightly. Scientists can measure the motion of the star by looking for a slight shift of the stellar spectrum, called the Doppler shift.

For 51 Peg:

1. Find the period, P, and half amplitude, v_{max}, of the star's motion.
2. How big is the shift in wavelength, $\Delta\lambda$ that corresponds to v_{max} (use $\lambda_0 = 6000$ Angstroms)? The typical width of a stellar spectral line at 6000 Angstroms is about 0.1 Angstroms. How does this shift compare to the width of the line?
3. Find the radius of the planet's orbit and its mass (figure out which equation to use based on the information you have and the information you want to calculate). Assume the orbit is edge-on, and the stellar mass is $1 M_{\mathrm{Sun}}$.
4. How does the mass of the planet compare to the mass of Jupiter? Note: $M_{\mathrm{J}} = 0.001 M_{\mathrm{Sun}}$.
5. Where would this planet be in the Solar System if it were orbiting the Sun? In other words, how does the size of its orbit compare to orbits of planets in the Solar System?

HD209458 was the first star found to have a transiting planet, meaning that the planet passes in front of the star from the perspective of the Earth during its orbit. Astronomers had already determined that HD209458 had a companion by Doppler-shift studies, and careful photometry revealed the slight decrease in the amount of starlight reaching us as the planet obscured part of the star.

For HD209458b:

1. Show that the fraction of star light blocked by the planet when it is in front of the star is $(R_{\mathrm{planet}}/R_{\mathrm{star}})^2$. Hint: if we could image them, both the planet and the star would look like circular disks in the sky, like the Moon or the Sun.
2. Use the graph to determine what fraction of the star light from HD209458 is blocked by the planet HD209458b?
3. What is the radius of the planet compared to the star $R_{\mathrm{planet}}/R_{\mathrm{star}}$ in this case?
4. If you assume the star has the same radius as the Sun, how does the radius of HD209458b compare to Jupiter. Note: The Sun's radius is 10 times larger than Jupiter's.

Finding the mass of an Exoplanet: the Doppler method

Finding the velocity

Light emitted by an object moving toward or away from the observer is shifted in wavelength. This is called the Doppler shift. Light from a receding object is shifted to longer wavelengths (redshifted), light from an approaching object is shifted to shorted wavelengths (blueshifted).

An object which is emitting light at a wavelength of λ_0 is moving with a velocity v (away from the observer) along the line of sight. The observer measures the wavelength λ. The relationship relating these quantities is:

$$\frac{\lambda - \lambda_0}{\lambda_0} = \frac{v}{c} \tag{1}$$

where c is the speed of light. Therefore a measured shift in wavelength can by used to infer a velocity.

Finding the mass

For a solar mass central body ($M_* = M_{\rm sun}$) with a low-mass companion ($M_c << M_*$), the period P and separation or orbital radius R are related by

$$P^2 = R^3 \tag{2}$$

with P measured in years and R in astronomical units (Kepler's Third Law).

The (centripetal) force required to hold a body of mass m traveling a speed v on a circular path of radius r is mv^2/r. In the star-planet system both objects travel in (roughly) circular orbits around their common center of mass. The force required to hold them in these orbits is provided by their mutual gravitational attraction, which is that same for the star acting on the planet or the planet acting on the star. Therefore the centripetal forces must be equal:

$$M_* \frac{v_*^2}{r_*} = M_p \frac{v_p^2}{r_p} = M_p \omega^2 r_p, \tag{3}$$

where $\omega = 2\pi/P$, P is the period, and r_* and r_p are the distances of the star and the planet from the center of mass, respectively. Note that $r_* = RM_p/(M_* + M_p) \approx RM_p/M_*$ and $r_p = RM_*/(M_* + M_p) \approx R$, where R is the distance from the star to the planet. So the equation can be rewritten

$$M_* \frac{v_*^2}{RM_p/M_*} = M_p \omega^2 R, \tag{4}$$

or,

$$M_*^2 v_*^2 = M_p^2 \omega^2 R^2. \tag{5}$$

We are now ready to solve for the mass of the planet. Taking the square root of both sides, and rearranging a little, we have

$$M_p = M_* \frac{v_*}{\omega R} = M_* \frac{v_*}{30,000\text{m/sec}\left(\frac{\text{R/1AU}}{\text{P/1yr}}\right)}. \tag{6}$$

Finally, since we know that $R = P^{2/3}$, we can write

$$M_p = M_* \frac{(v_*)}{(30,000\text{m/sec})} \frac{P}{1\text{yr}}^{1/3}. \tag{7}$$