

Greenhouse Effect & Habitable Zones

Lab # 7

Objectives: To model the effect of greenhouse gases on the radiative balance on Earth, and to think about factors that can affect the habitability of a planet.

Part 1: Radiative equilibrium

Quick recapitulation: Remember that every body emits radiation whose energy spectrum and intensity depend on its temperature. The relation between a body's luminosity per unit area and its temperature is given by the *Stefan-Boltzmann law*:

$$\mathcal{L} = \sigma T^4 \quad (1)$$

$$\sigma = 5.70 \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ K}^{-4} \quad (2)$$

The Sun and Earth both emit radiation, but since the Sun is much hotter (and bigger), it is also much more luminous. The Earth absorbs some of the Sun's radiation. The amount of radiation that the Earth receives from the Sun regulates its surface temperature. Since the average temperature of the globe is relatively stable, the Earth must radiate away the same amount of energy that it is receiving from the Sun. That is why we say that Earth is in *radiative equilibrium*.

1. Using the Stefan-Boltzmann law given above, write down the expression for the luminosity of Earth (L_{\oplus}) as a function of its radius (r_{\oplus}) and its temperature (T_{\oplus}).
2. Given a solar luminosity of L_{\odot} , an Earth's radius of r_{\oplus} and an Earth-Sun distance of d_{ES} , what amount of energy per unit time does the Earth receive from the Sun? (Remember, the Sun radiates its energy isotropically outward. Drawing a picture could help.)
3. Assuming Earth is in radiative equilibrium (how would you write this down mathematically?), derive the expression for Earth's temperature as a function of above parameters.
4. Calculate what the average surface temperature of the Earth should be using the actual values of the above parameters.
5. In reality, Earth does not capture all of the solar radiation that impinges it, but reflects about 30% of it back into space. Most of the reflection occurs from clouds, sand and snow/ice. The fraction of light that a planet reflects is called the planet's *albedo*. How would introducing an albedo (α) change your expression above for surface temperature, i.e. what is the new expression?
6. Recalculate Earth's surface temperature assuming it has an albedo of 0.3.
7. The actual measured average temperature at the surface of Earth is 15°C . How do your results compare to the actual value? If it is different, can you think of reasons why?

8. What would happen to the surface temperature of a planet that is NOT in radiative equilibrium with its parent star?

Part 2: Greenhouse effect

Greenhouse gases absorb *Infrared radiation*. Because of its low surface temperature Earth emits most of its energy in the infrared whereas the Sun, with its high surface temperature, emits mainly in the optical. Greenhouse gases thus act as a "blanket" over the surface of a planet by letting through energy coming in from the Sun, but capturing some of the energy reemitted by the surface of the Earth. This modifies the energy balance of the system and affects the surface temperature. To see how that works, let us pursue the blanket model, and simply assume that this "blanket" of greenhouse gases absorbs all of the radiation coming from the surface of the Earth (but that it is transparent to solar radiation). The gas molecules must then reemit this energy (otherwise the atmosphere would just heat up and evaporate!). They reemit also at infrared wavelengths, and equally in all directions. Half of the radiation will thus go back to the Earth, and half of it will escape to outer space.

1. Given that the layer of greenhouse gases reemits all of the energy it absorbs, write down the expression of its luminosity, L_L , as a function of L_{\oplus} , the luminosity of the Earth. (Very simple, do not think too hard on this one!)
2. Since the surface of the Earth also needs to be in radiative equilibrium, write down the expression for L_{\oplus} in terms of the energy it receives from the Sun as derived in question 5 of part 1, and of the luminosity of the greenhouse gas layer (remember half of the greenhouse gases radiation goes back to the Earth).
3. Solve the above two equations simultaneously for L_{\oplus} , by eliminating L_L .
4. Using the relation between luminosity and temperature that you derived in question 1 of part 1, write your new expression for the surface temperature.
5. Compute its value.
6. Redo your calculation assuming that the greenhouse gas layer absorbs respectively only 20, 40, 60 and 80% of Earth's radiated heat.
7. Make a plot of the surface temperature as a function of the fraction f_{abs} of L_{\oplus} absorbed by greenhouse gases. You should have six points: $f_{abs} = 0, 0.2, 0.4, 0.6, 0.8$ and 1.
8. In this model, how much greenhouse absorption is necessary in order to reproduce Earth's actual temperature?
9. What would be the effect of an increase in the concentration of greenhouse gases in the atmosphere?

Part 3: Habitable Zones

In this section we study the effect of the distance of a planet to its parent star on its habitability. Since all life as we know on Earth requires liquid water, the range of distances in

which a planet would have the right temperature to retain liquid water on its surface is called its *Habitable Zone*.

1. Using an albedo of 0.3 and the amount of greenhouse absorption found in question 8 of part 2, plot the average surface temperature of Earth as a function of distance to the Sun. Your plot should include at least distances in the range 0.5 to 2.0 AU. Also indicate where the orbits of Venus and Mars lie on your plot.
2. Assuming that liquid water is a prerequisite to life, measure from your plot, the habitability zone of Earth. (Remember, liquid water exists only at temperatures between 0°C and 100°C).
3. Given that the warmest regions of Earth can have yearly average temperatures as much as 20°C higher than the overall mean temperature, measure how far can the outer boundary be pushed back before the entire planet becomes a snowball and inhospitable to life.
4. The inner boundary of the Habitable Zone is on the other hand very uncertain because of the onset of a phenomenon called the *Runaway Greenhouse Effect* at temperature much below 100°C. A runaway greenhouse effect occurs when a planet is no longer able to compensate for the positive feedback loop in which high temperatures cause more evaporation, hence more water vapor in the atmosphere, hence more greenhouse effect since water vapor is also a greenhouse gas, hence higher temperatures, and so on. Once the runaway greenhouse effect is triggered, temperatures just soar upward until all water is evaporated. The exact temperature at which runaway greenhouse effect begins is unknown, but most models agree that on Earth, 40°C is a reasonable upper limit.

Assuming the above upper limit for the temperature at which Earth would remain habitable, what would be a more realistic inner boundary for the Earth's habitable zone?

5. Venus is an example of a planet that experienced a runaway greenhouse effect, and its surface temperature is now 470°C. In most other aspects, though, Venus is very similar to the Earth, and the two planets are thought to have been almost identical at the beginning of the Solar System. Yet, Venus suffered a very different fate. Is this observation consistent with your previous answer?
6. Some estimates suggest that Earth could be habitable up to distances of 1.7 AU, that is beyond Mars' orbit. Yet, Mars appears to be too cold to harbor life on its surface. Is that a contradiction? Or can you think of other factors affecting the habitability of both planets? Are there other differences between Mars and Earth than their distances to the Sun? (E.g.: The Moon does not host any life, yet it is at the same distance to the Sun than Earth is.)