Astronomy C3602: Homework #1

Due in class on Wednesday, February 6, 2008

Problem 1: The Ancient Greeks (20 points).

As mentioned in class, Aristarchus attempted to derive the distance to the Sun by triangulation between the Moon, Earth, and the Sun. He tried to observe exactly when the Moon was half-lit by the Sun. For this to happen, the Earth-Moon-Sun angle must be exactly 90 degrees. Aristarchus knew the Earth-Moon distance (from observing Earth's shadow on the Moon) to be 3.8×10^{10} cm.

(a) At the moment of half-illumination, Aristarchus mistakenly measured the Earth-Sun-Moon angle to be 3 degrees. What Earth-Sun distance did he infer? He assumed that the Moon orbits the Earth on a circle at constant speed. [**8 points**]

(b) Aristarchus inferred the Earth-Sun-Moon angle to be 3 degrees by measuring the time t_{13} elapsed from the 1st to the last (3rd) quarter of the Moon, and the time t_{31} elapsed from the 3rd to the 1st quarter. The moon takes $t_{13} + t_{31} = 29.5$ days to orbit the Earth (called a synodic month). What was Aristarchus' value for the difference $\Delta t = t_{13} - t_{31}$? We now know the correct Earth-Sun distance is 1 AU ($= 1.5 \times 10^{13}$ cm). What is the correct value for Δt? This is the accuracy to which Aristarchus would have had to identify the time of half-illumination. [**12 points**]

Problem 2: Star Counts, Dust, and the Size of the Universe (30 points).

Suppose the Universe consisted of a sphere of radius $R = 100$ kpc, filled uniformly with stars identical to the Sun, and a space density of $n = 1$ pc^{-3}. The Sun's luminosity is $L_\odot = 3.8 \times 10^{26}$ W, and 1 pc $= 3.1 \times 10^{16}$ m.

(a) What is the total number of stars? Derive the distribution $N(> F)$ of the number of stars whose flux as seen on Earth is brighter than F. Use W m^{-2} as the unit for F, and make a sketch of $\log N$ vs $\log F$. [**10 points**]

(b) Suppose that the universe is filled with dust that absorbs the light of stars. The effect of the dust is to dim the flux of stars. More distant stars are more strongly dimmed, since they are seen through a thicker slab of intervening dust. We will mimic this dimming as follows. The flux $F_0 = L/4\pi r^2$ that a star at a distance r would have in the absence of dust is (i) unaltered for stars closer than $r = r_0$, but (ii) reduced by a factor of $(r_0/r)^2$ to the observed value of $F = (r_0/r)^2 \times F_0$ for stars that are farther than $r = r_0$. Here $r_0 = 1$ kpc is a constant. What is the distribution $N(> F)$ in the presence of dust? Make a sketch of $\log N$ vs $\log F$ as above. [**10 points**]

(c) The measured star counts $N(> F)$ can be used for a rough estimate of the size of the universe. One can associate the flux F_e at which $N(> F)$ deviates from a pure power–law with the distance $r_e = (L_\odot/4\pi F_e)^{1/2}$ beyond which the space density of stars "thins out". Suppose someone who does not know dust exists (such as William Herschel in 1785) tries to use this method. Would (s)he obtain an overestimate or an underestimate for the size? By what factor, in the example (b) vs. (a) above? [**10 points**]

Problem 3: Olbers' Paradox (25 points).

Suppose the Universe consisted of a random, statistically uniform distribution of stars in space identical to the Sun, with a radius of $R = R_\odot = 7 \times 10^{10}$ cm, and a space density of $n = 1$ pc^{-3}. As shown in lecture, if such a Universe were infinitely old, the surface brightness of the sky as seen by an observer on Earth would be infinitely large.

(a) How far, on average, would you have to look in such a Universe until your line of sight struck the surface of a star? Express your answer in parsecs. [**13 points**]

(b) Show that if the Universe has a finite age, the surface brightness on Earth would be finite. What is this surface brightness, if the Universe is $t_0 = 15$ Gyr old? Express your answer in the units Wm^{-2} sr^{-1}. The Sun's luminosity is $L_\odot = 3.8 \times 10^{26}$ W. [**12 points**]

Problem 4: Hubble Expansion (20 points).

Hubble found that the universe is "expanding linearly", i.e. the recession velocity v of a galaxy is related to its distance from us as $v = H_0 r$. Here H_0 is called the Hubble constant. Convince yourself that such a linear expansion has the following properties:

(a) When the positions and velocities are considered relative to any other galaxy, the expansion still has the same linear form. [**7 points**]

(b) If the galaxies do not change their velocities, then the linear expansion law is preserved. [**7 points**]

(c) The linear velocity pattern could also be explained by a "explosion" that imparts different velocities to individual galaxies at some definite moment in the past (while such a "velocity sorting" could explain the present-day linear expansion, it would violate the cosmological principle). [**6 points**]

Problem 5: Feedback (5 points).

Please rank the previous three problems overall on a scale of 1-5 for (a) difficulty, (b) length, and (c) level of math involved. You will receive five points just for answering!