

Astronomy C3602: Homework #4

Due in class on Monday, April 7, 2008

Problem 1: Hydrogen and Helium Recombination (30 points).

We showed in class that hydrogen recombined at a redshift of approximately $z = 1200$, with recombination starting already at $z \sim 1600$ (see Figure 9.4 in Ryden).

(a) What was the ratio of the number density of hydrogen-ionizing photons (i.e. photons with energies $E \geq 13.6\text{eV}$) to the number density of hydrogen atoms at $z = 1600$? Explain why this ratio does not have to equal unity at recombination. [20 points]

(b) What was the ratio of the number density of helium-ionizing photons (i.e. photons with energies $E \geq 24.6\text{eV}$) to the number density of helium atoms at $z = 1600$? Does your answer imply that helium recombines before or after hydrogen? [10 points]

For this problem, assume the following: at the present day, baryons contribute a fraction $\Omega_b = 0.04$ of the critical density, of which $Y_{\text{H}} = 76\%$ by mass is hydrogen, and $Y_{\text{He}} = 24\%$ is helium. Assume a Hubble constant of $H_0 = 70 \text{ km/s/Mpc}$. The temperature of the cosmic microwave background (CMB) today is $T_0 = 2.725\text{K}$.

Problem 2: Alternative Origin of Helium and CMB (20 points).

We showed in class that the “hot big bang” model explains the origin and mass fraction of cosmic helium, as well as the presence of the CMB. Suppose, however, that the universe instead consisted originally of hydrogen alone, and that all of the observed helium (with a mass fraction of $Y_{\text{He}} = 24\%$) was created by fusion inside stars. How much energy was released per unit volume in the universe as a result of this helium fusion? Suppose that all of this energy is now in the form of radiation. Compare the resulting energy density to the present-day energy density of the CMB. Could the helium fusion account for the present-day CMB based on the energy density alone? Can you think of additional problems for this being the origin of the CMB? (Note: ${}^4\text{He}$ has a total binding energy of 28.3 MeV .)

Problem 3: Modified Nucleosynthesis (20 points).

In class, we estimated the maximum possible mass fraction of helium, $Y_{\text{He,max}}$ in the universe, by assuming that at the time of nucleosynthesis, all available neutrons were converted into ${}^4\text{He}$ nuclei. Suppose that the neutron decay time, $\tau_n = 890\text{s}$, was ten times shorter, $\tau_n = 89\text{s}$. What would be the value of $Y_{\text{He,max}}$? (hint: we showed that nucleosynthesis occurred approximately 200 seconds after neutron-proton freeze-out).

Problem 4: Reionization (30 points).

Recent observations by the *Wilkinson Microwave Anisotropy Probe (WMAP)* satellite showed that approximately 10% of the CMB photons suffered a scattering with an electron on their way from redshift $z = 1100$ to Earth. The explanation of this result may be that the universe is kept fully ionized by starlight at all redshifts below z_r . What is the value of z_r required to explain the electron scattering probability measured by *WMAP*? This would have to correspond to the epoch when the first stars were formed. For simplicity, assume a flat, $\Omega_m = 1$ universe with $\Omega_b = 0.04$ and $H_0 = 70$ km/s/Mpc. Compute z_r by assuming that all of the hydrogen is ionized, but all of the helium is neutral, throughout the entire interval $0 < z < z_r$. How does your answer change if you assume instead that all of the helium is in doubly ionized (He^{++}) form? (Note: the Thomson cross-section is $\sigma_T = 6.65 \times 10^{-25}$ cm².)