Astronomy C3602: Homework #5

Due in class on Monday, April 21, 2008

Problem 1: Inflation (30 points).

The best current measurement of the curvature of space (from the results of the *Wilkinson Microwave Anisotropy Probe (WMAP)* announced a few weeks ago) approximately indicates $\Omega_0 = 1.00 \pm 0.02$, where $\Omega_0 = \Omega_{\Lambda,0} + \Omega_{m,0} + \Omega_{r,0}$ is the present day energy density in units of the critical density. The remarkable closeness of Ω_0 to unity can be explained by an inflationary epoch in the early universe.

(a) Show that there is a 'fine-tuning problem' in the standard cosmological model, without inflation. In particular, if $\Omega_{\Lambda,0} = 0.68$, $\Omega_{m,0} = 0.3$, and $\Omega_{r,0} = 8.4 \times 10^{-5}$, then what was the value of the energy density, $\Omega(t) = \Omega_\Lambda + \Omega_m + \Omega_r$, at the following epochs: (i) when the energy densities of matter and the cosmological constant were equal? (ii) when the energy densities of matter and radiation were equal? (iii) at the Planck time ($t_P = 5 \times 10^{-44}$ seconds, when the scale factor, without inflation, would be $a_P = 2 \times 10^{-32}$) ? [**10 points**]

(b) How many e-foldings of inflation would be needed to solve this fine-tuning problem? Assume that the inflationary epoch starts at some initial time t_i during the radiation-dominated epoch, and that $1 - \Omega(t) \sim 1$ before inflation. Assume further that inflation lasts for N Hubble times, so that it ends at $t_f = (N+1)t_i$, with $t_f = 4 \times 10^{-36}$ seconds. Compare your answer to $N = 60$ mentioned on page 200 of Ryden. Does the 10-fold increase in precision from $\mid 1 - \Omega_0 \mid \leq 0.2$ (available when the textbook was written) to $\mid 1 - \Omega_0 \mid \leq 0.02$ (available from *WMAP* now) necessitate many more e-foldings? [**10 points**]

(c) Since the present-day energy density is dominated by a cosmological constant, the Universe is also 'inflating' at the present time. What will be the value of the scale factor, $a(t)$ when the age of the universe 137 Gyr, or 10 times its current value (in estimating this value, ignore the contribution from matter, and assume a flat universe with cosmological constant only). What will be the value of the energy density, $\Omega(t)$ at this epoch ? Does the fine-tuning problem become better or worse? [**10 points**]

Problem 2: Jeans Mass (20 points)).

As discussed in class, in a perturbation theory for an expanding fluid with density ρ and sound speed v_s, that the Jeans wavelength $\lambda_J = 2\pi/k_J$, with $k_J = (4\pi G\rho/v_s^2)^{1/2}$, separates oscillating from growing sinusoidal perturbations. The Jeans mass M_J is defined as the amount of mass enclosed in a sphere of radius $\lambda_J/2$.

(a) Derive the Jeans mass for baryons just after decoupling at $z = 1100$. Assume that the baryons at this redshift have the same temperature as the CMB, $T_b = T_{\rm cmb} = 2.7(1+z)$K, and also assume the parameters $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, $\Omega_b = 0.04$, and $H_0 = 72$ km/s/Mpc. How does the Jeans mass evolve with redshift just after $z = 1100$? [**15 points**]

(b) The temperature of the baryons is locked by Compton scattering to that of the CMB only down to a redshift of $z = 150$. After this redshift, the universe is too dilute to efficiently couple the baryon and the CMB temperatures, and the baryons cool adiabatically, $T_b \propto \rho_b^{2/3}$. What is the baryon Jeans mass, as a function of redshift, at $z < 150$? [**5 points**]

Problem 3: Galaxy Formation (30 points).

In this problem, we show that a measurement of the abundance and masses of galaxies, together with a measurement of the *r.m.s.* density fluctuations on galactic scales, can be used to estimate when galaxies formed.

(a) The observed number density of bright galaxies today is $n_g = 10^{-3}$ Mpc^{-3}. Each galaxy has a total (dark matter + baryon) mass of 10^{12} M$_\odot$. What fraction of the total mass of the universe has collapsed into these galaxies? Assume $\Omega_m = 0.3$. [**10 points**]

(b) The distribution of the values of the overdensity $\delta = \Delta\rho/\rho$ at different locations is given by a Gaussian with an *r.m.s.* amplitude of $\sigma = \langle\delta^2\rangle^{1/2}$. Suppose each spherical region that has an overdensity of $\delta \geq \nu_g\sigma$ grows into a galaxy by the present epoch. Find the value of ν_g. (Hint: you will need erfc, the complementary error function). [**10 points**]

(c) Observations indicate that if the primordial density fluctuations were evolved to redshift $z = 0$ using linear theory, they would have $\sigma(M) = 3$ on a scale of 10^{12} M$_\odot$. From the spherical collapse model, the collapse redshift z_g can be identified as the redshift at which the amplitude of the initial overdensity, evolved forward using the linear solution, would be $\Delta\rho/\rho = 1.69$. Compute the galaxy formation redshift z_g assuming (for simplicity) that the density perturbations grow as $\Delta\rho/\rho \propto (1+z)^{-1}$. [**10 points**]

Problem 4: Present-Day Structure Formation (20 points).

Large scale structure observations indicate that on scales of 11 (comoving) Mpc, the *r.m.s.* amplitude of density fluctuations at the present time is $\sigma_{11} = 1$. This means that spheres with this radius are turning non-linear in the present-day universe.

(a) What is the mass of these objects? What astronomical objects do they correspond to? Assume $\Omega_m = 0.3$. [**10 points**]

(b) Can you think of a way of testing whether these objects are indeed in the process of forming today? [**10 points**]