Astronomy C3602: Problem Set #6

Due in class on Monday, May 5, 2008

Problem 1: Isothermal Gas Sphere (40 points).

Consider an overdense initial perturbation that separates from the expansion of the universe, turns around, and collapses to form a spherically symmetric, virialized object.

(a) The collapsed gas is shock–heated to the virial temperature $T_{\rm vir}$. Assume that $T_{\rm vir}$ is a constant and independent of radius (this is generally a good assumption because conduction would quickly smooth out spatial temperature gradients). Show that the density profile must then have the scaling $\rho = B/r^2$ for the system to be in equilibrium; i.e. for the pressure gradient force to balance the gravitational force. Find an expression for the coefficient B as a function of $T_{\rm vir}$. [**15 points**]

(b) Assume that the object collapsed at redshift z, it consists of pure (ionized) hydrogen gas (ignore helium and dark matter), and has a total mass $M_{\rm vir}$, radius $r_{\rm vir}$, and a mean density $\bar{\rho} = 18\pi^2\rho_c$. Here $\rho_c = 3H^2/(8\pi G)$ is the critical density at the redshift of collapse, and $H(z)$ is the Hubble parameter at z. Derive the virial temperature for a typical galaxy with $M_{\rm vir} = 10^{12}{\rm M}_\odot$, $z = 2$, and $H_0 = 72$ km/s/Mpc. How does the virial temperature scale with $M_{\rm vir}$, z, and H_0 ? For simplicity, assume a flat universe with $\Omega_m = 1$. [**20 points**]

Problem 2: Angular Momentum (35 points).

Consider a uniform sphere of mass M and radius R, spinning with a total angular momentum J. The spin parameter $\lambda_i = (JE^{1/2})/(GM^{5/2})$ measures the ratio of the angular velocity corresponding to J and the angular velocity required to provide full centrifugal support. Here $E = (3/5)GM^2/R$ is the total energy.

(a) Suppose the sphere is compressed by a factor of f_c into a new uniform sphere with radius $R_c = f_c R$. Assuming that angular momentum is conserved, find the value of the spin parameter λ_c after the compression, as a function of f_c and λ_i. [**5 points**]

(b) Suppose that only a fraction $f_b = \Omega_b/(\Omega_m + \Omega_b)$ of the original sphere (by mass) is compressed by a factor of f_c into a new uniform sphere with radius $R_c = f_c R$, with the remaining fraction $1 - f_b = \Omega_m/(\Omega_m + \Omega_b)$ staying in its original configuration as a sphere of radius R. (This is a simplified picture of baryons cooling and condensing inside a stationary dark matter halo). Assume that the compressed material contains a fraction f_b of the total initial angular momentum. Find the value of the spin parameter λ_c of the compressed material, as a function of f_b, f_c, and λ_i. [**15 points**]

Problem 3: Assembly of Milky Way (25 points).

In class, we derived $f(M,z)$, the fraction of the total mass of the Universe that is contained inside collapsed objects with masses M or larger at redshift z; we found $f = \mathrm{erfc}\left[\delta_c(z)/\sqrt{2}\sigma(M)\right]$. Here δ_c is the overdensity threshold for collapse in the spherical collapse model. We argued that a similar expression, substituting $\delta_c(z)$ by $\delta_c(z_1) - \delta_c(z_0)$, and $\sigma(M)$ by $\sqrt{\sigma(M_1)^2 - \sigma(M_0)^2}$, gives the fraction $f(M_1, z_1; M_0, z_0)$ of the mass of a collapsed object with mass M_0 at redshift z_0 that was locked up in collapsed "progenitor" objects of mass M_1 or larger at the earlier redshift $z_1 > z_0$.

(a) Near $M = 10^{12}\mathrm{M}_\odot$, the r.m.s. density fluctuation in spheres of mass M is given approximately by $\sigma(M) = 3(M/10^{12}\mathrm{M}_\odot)^{-1/6}$. Use this expression to calculate the fraction of the mass in the universe that resides in galaxies with masses similar to that of the Milky Way ($M = 10^{12}\mathrm{M}_\odot$) or higher. [**5 points**]

(b) What fraction of the present mass of the Milky Way was contained in globular cluster sized clumps ($M = 10^6\mathrm{M}_\odot$ or higher) at redshift $z = 3$? [**10 points**]